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Brown Dwarfs:

If electrons become degenerate at the center of the protostar before the temperature becomes high enough to ignite nuclear reactions at a significant rate, we will have brown dwarfs. These are failed stars that do not go further contraction due to degeneracy pressure of electrons, but do not shine as a result of nuclear burning either.

The brown dwarfs are fully convective from the center to the photosphere. Pressure is provided essentially by non-relativistic degenerate electrons and an ideal gas of ions. For an ideal monatomic gas we have:

$$P_{ideal} = \frac{2 \rho k_B T}{m_u} \quad (100\% \text{ ionization})$$

While for a non-relativistic degenerate electron gas:

$$P_{deg} = (3\pi^2)^{2/3} \left(\frac{\hbar^2}{m_e}\right)\left(\frac{\rho}{m_u \mu_e}\right)^{5/3} \simeq 10^{13} \rho^{5/3}$$

Here $m_u \nu_e$ is the average particle masses per free electron.

The degeneracy pressure dominates if:

$$T < 3\times10^{5}\left(\frac{\rho}{1g\ cm^{-3}}\right)^{2/3} K$$

We define a parameter:

$$\xi = \frac{P_{ideal}}{P_{deg}} = 8\times10^{-6}\, T\, \rho^{-\frac{2}{3}} \propto \left(\frac{k_B T}{E_F}\right)$$

Where $E_F \propto \rho^{2/3}$ is the Fermi energy. Then:

$$P \simeq 10^{13}\, \rho^{5/3} f(\xi)\ dyn\ cm^{-2}$$

Where:

$$f(\xi) \rightarrow \begin{cases} 1+\xi & \xi \ll 1 \quad \text{(degenerate limit)} \\ 2\xi & \xi \gg 1 \quad \text{(non-degenerate limit)} \end{cases}$$

For a convective star energy transport occurs via adiabatic gradient, and hence entropy remains constant throughout the star. Lets find entropy per nucleon $\frac{S}{k_B}$. We start with a monatomic ideal gas. According to the first law of thermodynamic:

$$dU = T\,ds - P\,dv$$

Here U, S, v represent internal energy, entropy, volume per nucleon respectively.

We have:

$$U = \frac{3}{2} k_B T \quad , \quad \frac{dv}{v} = \frac{-d\rho}{\rho} \quad , \quad Pv = k_B T$$

Thus:

$$d\left(\frac{s}{k_B}\right) = \frac{1}{T} d\left(\frac{U}{k_B}\right) + \frac{P}{k_B T} dv = \frac{3}{2}\frac{dT}{T} + \frac{dv}{v} = \frac{3}{2}\frac{dT}{T} - \frac{d\rho}{\rho}$$

$$\Rightarrow \frac{s}{k_B} = A \ln\left(\frac{T}{\rho^{\frac{2}{3}}}\right) - B$$

Where A, B are constants.

For a degenerate plasma, with Coulomb interactions included, numerical studies suggest that:

$$\frac{s}{k_B} = 2.2 \ln\left(\frac{T}{\rho^{0.63}}\right) - 11.6$$

Therefore $\frac{T}{\rho^{2/3}}$ is constant to a very good approximations for convective stars. Note that $T \propto \rho^{\frac{2}{3}}$ results in:

$P = k\rho^{\frac{5}{3}}$

This is a polytrope with $n = \frac{3}{2}$, which we discussed earlier (see Lecture 10). The solution of Lane-Emden equation for $n = \frac{3}{2}$ gives:

$$\underbrace{\rho_c}_{\text{central density}} = 5.991\,\bar{\rho} \quad , \quad R = \underbrace{2.8 \times 10^{9} \left(\frac{M_\odot}{M}\right)^{\frac{1}{3}}}_{R_0} f(\xi)$$

Recall that:

$$\xi \propto T\rho^{-\frac{2}{3}} = \text{Const.} \Rightarrow \xi \propto T M^{-\frac{2}{3}} R^{2} = 3 \times 10^{-9} \underbrace{T_c}_{\text{central temperature}} \left(\frac{M_\odot}{M}\right)^{\frac{4}{3}} f^{2}(\xi)$$

The central temperature is then found to be:

$$\boxed{T_c = 3 \times 10^{8} \left[\frac{\xi}{f^{2}(\xi)}\right] \left(\frac{M}{M_\odot}\right)^{\frac{4}{3}} K}$$

The central temperature is maximized at $R = 1.75\,R_0$.

For much larger R the star is non-degenerate, while for smaller R degeneracy pressure is dominant hence temperature drops.

For the pure Hydrogen case ($X \simeq 1$) we have:

$$T_{c,max} = 5.4 \times 10^7 \left(\frac{M}{M_\odot}\right)^{\frac{4}{3}} K$$

While for the realistic case, with $X \simeq 0.7, Y \simeq 0.3$, we have:

$$T_{c,max} = 8 \times 10^7 \left(\frac{M}{M_\odot}\right)^{\frac{4}{3}} K$$

The star fails and becomes a brown dwarf if Hydrogen burning at the time T_c has its maximum value is not enough to prevent gravitational collapse. The mass for which this happens can be found by comparing the luminosity from Hydrogen burning and the rate of gravitational energy release from contraction. The former is given by:

$$L_{pp} \simeq \varepsilon_{pp}(T_c) \, \rho_c \, R_c^3 \quad , \quad R_c \simeq 0.38 R \quad (\rho_{(r=R_c)} \simeq \tfrac{1}{2}\rho_c)$$

$$\varepsilon_{pp}(T) \simeq 2.4 \times 10^6 \, \rho \, X^2 \, T_6^{-\frac{2}{3}} \quad \exp\left[-33.8 \, T_6^{-\frac{1}{3}}\right] \text{ erg } g^{-1} s^{-1}$$

Here $T_6 \equiv \frac{T}{10^6 K}$. For $X=1$ and $T_c = T_{c,max}$ we have:

$$\boxed{\begin{aligned} L_{pp,max} &= 4\times10^{36}\left(\frac{M}{M_\odot}\right)^3 F_{(T_c)} \ \text{erg s}^{-1} \\ F_{(T)} &= 2.4\times10^{6}\, T_6^{-\frac{2}{3}} \exp\left[-33.8\, T_6^{-\frac{1}{3}}\right] \end{aligned}}$$

On the other hand, the rate for gravitational energy release can be estimated as:

$$L_{grav} \sim \frac{\Omega}{\tau_U}$$

Where τ_U is the age of the universe and:

$$\Omega = \frac{3}{7}\frac{GM^2}{R_{(T_c,max)}}$$

This is the gravitational potential energy of a polytrope with $n = \frac{3}{2}$ at the radius for which T_c assumes its maximum value.

It is found that $L_{pp,max} > L_{grav}$ if $\underline{\underline{M > 0.085 M_\odot}}$.

The actual value of the prefactor is close to this, and is found from more sophisticated modelling.

Therefore gas clouds with a mass $\underline{M < 0.085 M_\odot}$ end up as brown dwarfs.

It is also possible to find some general results about the cooling of brown dwarfs. The outmost layers of brown dwarfs are cool enough to have Hydrogen in molecular form, instead of atoms. The pressure at the photosphere is (see pages (121) and (122)):

$$P_{ph} \simeq \frac{2}{3} \frac{g}{\kappa}$$ (κ: opacity, g: gravitational acceleration at the surface)

For molecular Hydrogen we have:

$$P_{H_2} = \frac{\rho_{ph} k_B T_e}{2 m_u}$$

Then, setting $P_{ph} = P_{H_2}$, we find:

$$\rho_{ph} = \frac{4}{3} \frac{G M m_u}{k_B T_e R^2 \kappa}$$

For a diatomic gas:

$$\frac{s}{k_B} = A_1 \ln\left(\frac{T}{\rho^{n_1}}\right) - B_1$$

In the ideal gas $A = 2.5$, $n_1 = 0.4$. A more precise calculation shows that $A \simeq 1.3$, $n_1 \simeq 0.42$, $B_1 \simeq 3.0$.

Since $\frac{s}{k_B}$ is constant throughout the brown dwarf, we have $s_{ph} = s_c$, where:

$$\frac{s_{ph}}{k_B} = 1.8 \ln T_e + 1.08 \ln\left(\frac{R}{R_0}\right) + 0.54 \ln\left(\frac{\kappa}{10^{-2}}\right) - 0.82 \ln\left(\frac{M}{M_\odot}\right) - 4.6$$

$$\frac{s_c}{k_B} = 2.2 \ln T_c - 2.78 \ln\left(\frac{M}{M_\odot}\right) + 4.17 \ln\left(\frac{R}{R_0}\right) - 28.0$$

This gives us T_e in terms of T_c:

$$T_e = 2 \times 10^{-6}\, T_c^{1.22} \left(\frac{M}{M_\odot}\right)^{-1.05} \left(\frac{R}{R_0}\right)^{1.7} \left(\frac{\kappa}{10^{-2}}\right)^{-0.3}$$

The importance of this relation is that the luminosity $L = 4\pi R^2 T_e^4$ is entirely determined by T_c. This can be used to find an equation for the cooling rate in terms of mass, central temperature, and photospheric opacity. The initial cooling of the brown dwarfs is found to follow $L \propto t^{-1.3}$.